A Dynamical Torsion Field Theory of Gravity and Cosmology

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January 2026

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DOI: 10.5281/zenodo.18280119

Abstract

We present a covariant torsion–phase field theory in which gravitational acceleration and cosmological expansion emerge from the dynamics of a single geometric degree of freedom associated with expansion–compaction phase misalignment. Spacetime geometry is described by a Lorentzian metric with Levi–Civita connection, while torsion is promoted from a subsidiary geometric correction to an independent dynamical field τ_μ whose gradients generate effective gravitational acceleration and whose homogeneous dynamics drive large-scale expansion.

Starting from a minimal generally covariant action, we derive the coupled field equations, stress–energy tensor, and sourced torsion dynamics. Introducing a compaction current, we show that the weak-field, non-relativistic limit reduces to a Poisson equation for the torsion potential, yielding $\tau_0 = GM/r$ as the unique exterior solution for isolated masses and recovering the Newtonian inverse-square law and Schwarzschild weak-field metric as derived consequences rather than imposed ansätze. The theory reproduces all standard weak-field and Solar-System tests and remains empirically indistinguishable from General Relativity in currently probed regimes.

In the homogeneous cosmological limit, a time-like torsion background naturally generates an effective dark-energy sector with equation of state $w \simeq -1$ in a stable quasi-static regime, providing a unified geometric origin for local gravitational acceleration and global cosmic expansion. We formulate the linear perturbation framework, establish conditions for local and cosmological stability, and identify the propagating mode structure and observational channels through which deviations from General Relativity may arise.

The strong-field sector is parameterised phenomenologically, allowing controlled departures near compact objects and defining targets for future observational tests. An explicit perturbative roadmap is provided, outlining the programme required to fully characterise

the dynamical spectrum and confront the theory with precision data. This work establishes the foundational field-theoretic structure of the torsion–expansion framework and provides a coherent platform for extensions to strong-field gravity, cosmology, and quantum regimes.

Contents

1 Introduction

Gravity is traditionally understood either as a force mediated by mass, as in Newtonian theory, or as the manifestation of spacetime curvature induced by energy–momentum, as in General Relativity (GR). While GR has been extraordinarily successful across a wide range of scales, its geometric description relies exclusively on curvature and treats torsion, when present at all, as a non-propagating correction sourced by intrinsic spin in Einstein–Cartan extensions.

At the same time, modern cosmology requires the introduction of additional components—dark energy and dark matter—to account for large-scale observations, suggesting that the geometric description of spacetime may be incomplete. In particular, GR provides no intrinsic link between local gravitational acceleration and global cosmological expansion; these phenomena are introduced as separate sectors of the theory.

In this work we present a dynamical torsion field theory in which gravity emerges from gradients of a torsion–phase field associated with the geometric tension between expansion and compaction. Torsion is promoted from a subsidiary geometric correction to a genuine dynamical field capable of sourcing gravitational acceleration, reproducing the weak-field predictions of GR while naturally extending to cosmological scales.

The central result is that local gravitational acceleration can be reinterpreted as a kinematic effect of shell-wise expansion driven by torsion gradients, while homogeneous torsion dynamics generate an effective dark-energy sector in cosmology. The theory is minimal, covariant, and empirically indistinguishable from GR in tested regimes, while offering a unified geometric origin for gravity and cosmic acceleration.

Scope and Limitations. The present work establishes the foundational field-theoretic structure of the torsion–expansion framework and demonstrates its consistency with known weak-field, Solar-System, and cosmological limits. However, the analysis is intentionally restricted to the construction of the covariant action, the identification of sourced field equations, the derivation of the Newtonian limit, and the formulation of the perturbative framework. A complete mode-by-mode perturbation analysis, explicit strong-field solutions of the coupled system, and detailed confrontation with observational datasets lie beyond the scope of this paper and are reserved for dedicated companion studies. Accordingly, the strong-field sector is parameterised phenomenologically, and the perturbation results are presented at the level of stability structure and programme definition rather than full spectral classification.

Clarification on Terminology. The field τ_μ introduced in this work is *not* Cartan torsion and does not modify the affine connection or the rules of parallel transport. Throughout, the Levi–Civita connection is assumed. The field τ_μ is instead an independent, covariant *torsion–phase vector field* encoding shell–wise rotational phase misalignment between expansion and compaction within the ECT framework [1].
While the term *torsion* is retained to emphasise its geometric origin in rotational stress and phase winding, τ_μ is mathematically a vector degree of freedom with Maxwell–Proca–like dynamics. To avoid confusion, we will occasionally refer to τ_μ as a *torsion–phase field*, *torsion–like vector*, or *phase–torsion field.* No claim is made that this field corresponds to Cartan torsion, teleparallel torsion, or any modification of the spacetime connection.

The conserved compaction current J^μ is treated in this work as an *effective, emergent* source encoding the leading–order influence of non–relativistic matter on the torsion–phase field. It is not introduced as a new fundamental degree of freedom, but as a macroscopic current arising from coarse–grained matter distributions in the weak–field regime. At leading order, J^μ is independent of the torsion self–energy and is not algebraically identified with the full stress–energy tensor $T^{\mu\nu}$, though it is consistent with it through the Einstein equations and conservation laws. A microscopic derivation of J^μ from matter fields, and its precise relation to $T^{\mu\nu}$ beyond the Newtonian limit, are deferred to subsequent papers in the ECT programme. This work builds directly on the geometric formulation of gravity introduced in [2], where gravitational phenomena emerge from the interplay of expansion, compaction, and torsion within a unified ECT framework. The present paper refines that construction by reinterpreting local gravitational acceleration as surface expansion of compacted matter shells.

2 Geometric Framework

We begin by introducing the minimal geometric ingredients required for the theory. The framework is built on three interacting geometric tendencies:

- **Expansion**, representing outward geometric unfolding of spacetime,
- **Compaction**, representing inward geometric resistance associated with matter density,
- **Torsion**, representing phase misalignment between expansion and compaction.

Rather than treating torsion as an antisymmetric correction to the affine connection sourced by spin, we encode torsion through a covariant *torsion–phase potential* τ_μ. This field captures the local phase tension between expansion and compaction and acts as the primary dynamical quantity of the theory.

The spacetime geometry is described by a Lorentzian metric $g_{\mu\nu}$ with Levi–Civita connection ∇_μ. All curvature quantities are defined in the standard way from this connection. Torsion does not modify the connection directly; instead, it contributes dynamically through its stress–energy and its coupling to curvature.

The scalar quantity

$$s \equiv \tau_\mu \tau^\mu \tag{1}$$

measures the local torsion magnitude and determines the self-interaction of the torsion field.

This approach ensures that the theory remains fully covariant and reduces smoothly to General Relativity in the limit of vanishing torsion. A detailed derivation of the ECT gravitational field equations, torsion potential, and their equivalence to the weak-field limit of general relativity is given in [2]; here we summarise only the elements required for the surface-expansion reinterpretation developed in this work.

Torsion-Phase Field and Cartan Torsion. It is important to clarify the geometrical status of the torsion–phase field introduced in this work. The spacetime connection is taken throughout to be the Levi–Civita connection of the metric, and no antisymmetric component of the affine connection is introduced. In this sense, the present theory is not an Einstein–Cartan or teleparallel formulation and does not employ Cartan torsion as a fundamental geometric property of the connection. Instead, τ_μ is an independent covariant field encoding effective torsion-phase dynamics associated with expansion–compaction misalignment. The term "torsion" is therefore used here in the sense of a dynamical phase and geometric tension field, rather than as a modification of parallel transport. This distinction ensures that the standard differential geometry of General Relativity is preserved while allowing additional torsion-phase degrees of freedom to propagate dynamically.

Vector–tensor and torsion–based modifications of gravity have been explored in several distinct contexts, including Einstein–Æther theories [4], generalized (Proca) vector gravity [5], cosmological vector dark energy models [6], and teleparallel or Cartan–torsion formulations of gravity [7]. The present work differs from these approaches in that the torsion field is treated as a dynamical geometric phase variable with a Maxwell–type kinetic structure and a scalar potential $V(s)$, from which effective masses, stability properties, and cosmological behaviour emerge dynamically rather than being imposed by constraint or operator tuning. In particular, no unit–norm constraint, preferred–frame condition, or spin–density source is assumed. Einstein–Æther theories [4], generalized Proca gravity [5], cosmological vector dark energy [6], and torsion-based gravity frameworks [7] provide useful points of comparison. The algebraic structure underlying expansion, compaction, and torsion is formalised as a minimal three–generator Lie algebra in [3], providing the operator backbone for the present geometric formulation.

3 Torsion Field Dynamics

The dynamics of the torsion field are defined through an action principle. We take the total action to be

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(s)\right], \qquad F_{\mu\nu} \equiv \nabla_\mu \tau_\nu - \nabla_\nu \tau_\mu, \quad s \equiv \tau_\mu \tau^\mu. \tag{2}$$

where R is the Ricci scalar and $V(s)$ is a scalar potential encoding self-interaction and compaction effects.

Variation with respect to τ_ν yields the torsion field equation

$$\nabla_\mu F^{\mu\nu} - 2V'(s)\tau^\nu = 0. \tag{3}$$

where $V'(s) = dV/ds$. This equation shows that torsion propagates dynamically, couples directly to spacetime curvature, and acquires an effective mass through the potential term.

Variation with respect to the metric produces the modified Einstein equations

$$G_{\mu\nu} = 8\pi G\, T^{(\tau)}_{\mu\nu}, \tag{4}$$

with torsion stress–energy tensor

$$T^{(\tau)}_{\mu\nu} = F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + 2V'(s)\tau_\mu\tau_\nu - g_{\mu\nu}V(s). \tag{5}$$

This structure is fully covariant, dimensionally consistent, and free of higher-derivative instabilities. In the limit $\tau_\mu \to 0$, the theory reduces exactly to vacuum General Relativity.

In the following sections we show how specific torsion configurations reproduce Newtonian gravity and relativistic weak-field effects, while homogeneous torsion dynamics generate an effective cosmological dark-energy sector.

Stress–energy variation. Although the torsion field strength is defined using the Levi–Civita covariant derivative,

$$F_{\mu\nu} \equiv \nabla_\mu \tau_\nu - \nabla_\nu \tau_\mu, \tag{6}$$

the Christoffel-symbol contributions cancel identically in the antisymmetrisation (torsion-free connection),

$$F_{\mu\nu} = \partial_\mu \tau_\nu - \partial_\nu \tau_\mu. \tag{7}$$

Consequently, $F_{\mu\nu}$ is independent of the metric connection, and the metric variation of the kinetic term proceeds exactly as in Maxwell–Proca theory: only the index-raising operations and the factor $\sqrt{-g}$ contribute. The potential sector varies through $s = \tau_\mu \tau^\mu = g^{\mu\nu}\tau_\mu\tau_\nu$, producing the additional $2V'(s)\tau_\mu\tau_\nu - g_{\mu\nu}V(s)$ terms. The resulting stress–energy tensor is therefore given by Eq. (5) without additional connection-variation contributions. The antisymmetric torsion field strength defined here represents the classical, geometric limit of the torsion gauge construction developed in [8], where analogous Maxwell–type field structures are shown to reproduce QED and QCD scattering corrections within the ECT framework.

Definition 1 (ECT Phase Consistency Rule)**.** Let $\theta(x)$ denote the *dimensionless torsion phase variable* associated with the torsion–phase sector of the ECT framework, defined on a spacetime domain $\mathcal{M}$ with the property that θ is single-valued wherever the field configuration is regular. We say that a configuration satisfies the *ECT Phase Consistency Rule* if:

1. **Global phase coherence (quantized winding).** For any closed loop $\gamma \subset \mathcal{M}$ that lies entirely within a simply connected, regular region where θ is defined continuously, the total phase winding is quantized,

$$\oint_\gamma \nabla_\mu \theta \, d\ell^\mu \;=\; 2\pi n, \qquad n \in \mathbb{Z}. \tag{8}$$

2. **Local phase events (interface classes).** Across localized interfaces Σ (e.g. shell boundaries, matching layers, or defect surfaces), the phase increment is restricted to one of the following classes,

$$\Delta\theta\big|_\Sigma \in \left\{ 0,\ \frac{\pi}{2},\ \pi,\ \text{continuous drift} \right\}, \tag{9}$$

where the admissible class is determined by the interface type and dynamical regime.

3. **Orientation inversion.** A π-shift is interpreted as an *orientation/sign inversion* of the torsion phase, but the theory does not assume that all phase events are π-events.

This rule is intended as an *operational consistency constraint*: it restricts the allowed global and interfacial phase behaviour without imposing any microscopic quantization of local field amplitudes.

Lemma 1 (Topological invariance of winding)**.** *Assume θ is continuous and single-valued on an open set $U \subset \mathcal{M}$, and let $\gamma_0, \gamma_1 \subset U$ be two closed loops that are homotopic within U. Then the winding number*

$$n(\gamma) \;\equiv\; \frac{1}{2\pi} \oint_\gamma \nabla_\mu \theta \, d\ell^\mu \tag{10}$$

is invariant under continuous deformations of the loop within U; i.e. $n(\gamma_0) = n(\gamma_1)$. Moreover, $n(\gamma) = 0$ for any loop γ that is contractible to a point in U.

Proof. Because θ is single-valued and continuous on U, the 1-form $\omega \equiv d\theta$ is exact on U. For a contractible loop γ bounding a surface $S \subset U$, Stokes' theorem gives

$$\oint_\gamma d\theta \;=\; \int_S d(d\theta) \;=\; \int_S 0 \;=\; 0,$$

so $n(\gamma) = 0$. If γ_0 and γ_1 are homotopic in U, then $\gamma_1 - \gamma_0$ is the boundary of a 2-chain contained in U, and the same argument implies $\oint_{\gamma_1} d\theta = \oint_{\gamma_0} d\theta$. Hence $n(\gamma)$ is invariant under loop deformations within U. Nonzero n therefore requires that the homotopy between loops crosses a region where θ fails to be globally single-valued (e.g. a defect core or phase interface), consistent with interpreting n as a topological label of shell/defect structure. □

Remark 1 (Scope). Equations (8)–(9) constrain global coherence and interface behaviour of the torsion phase. They do not, by themselves, imply quantization of local energy levels or Standard Model charges. Instead, they provide a robust topological classifier for shell-like configurations and a disciplined way to connect distinct phase-event classes to distinct observational channels in later phenomenology.

4 Spinor–Sheet Lagrangian for Light and the Hopf Map

4.1 Two-sheet spinor representation of the light field

We model the "1995 two orthogonal oscillatory sheets" picture of light as a pair of Weyl spinor fields carrying a locked relative phase: a forward-oriented sheet ψ_+ and a backward-oriented sheet ψ_-, coupled at a fixed right angle in internal space. Concretely, let

$$\psi_+(x) \text{ and } \psi_-(x) \tag{11}$$

be two left-handed Weyl spinors (or one Dirac spinor decomposed into two Weyl components). We enforce the orthogonal-sheet condition as an $SU(2)$ constraint on a *normalized* two-component complex field

$$z(x) \equiv \begin{pmatrix} z_1(x) \\ z_2(x) \end{pmatrix} \in \mathbb{C}^2, \qquad z^\dagger z = 1, \tag{12}$$

by identifying

$$z(x) \;\sim\; \frac{1}{\sqrt{\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-}} \begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix}. \tag{13}$$

The *relative phase* between ψ_+ and ψ_- is the internal "torsion twist" that will become the Hopf fiber coordinate.

4.2 Spinor Lagrangian coupled to the ECT torsion–phase sector

Your torsion–phase field already obeys the dynamical structure (vector field τ_μ, field strength $F_{\mu\nu} = \partial_\mu \tau_\nu - \partial_\nu \tau_\mu$, and locking potential $V(s)$ with $s = \tau_\mu \tau^\mu$) used throughout Papers II–III. We extend the action by an $SU(2)$ spinor sector whose *internal connection* is built from a sheet-coupling gauge field $A_\mu = A^a_\mu \sigma^a/2$. The minimal coupled Lagrangian density is

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(s)}_{\text{ECT torsion–phase sector}} + \underbrace{i\,\psi^\dagger \bar{\sigma}^\mu D_\mu \psi - \mu\,\psi^\dagger \psi}_{\text{spinor sheet sector}} + \underbrace{\frac{\kappa}{2}\,\tau_\mu\, J^\mu_{\text{sheet}}}_{\text{sheet–torsion coupling}}, \tag{14}$$

$$D_\mu \psi \equiv (\partial_\mu - ig\,A_\mu)\psi, \qquad \psi \equiv \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \tag{15}$$

where $\bar{\sigma}^\mu = (\mathbf{1}, -\vec{\sigma})$ in a mostly-plus metric convention, μ is an effective mass-scale (set $\mu = 0$ for strictly massless propagation), g is the $SU(2)$ coupling, and κ controls how the ECT torsion vector biases the relative sheet current. The *sheet current* is the $SU(2)$ Noether current

$$J^\mu_{\text{sheet}} \equiv \psi^\dagger \bar{\sigma}^\mu \psi \quad \text{(and optionally its isovector version } J^{\mu a}_{\text{sheet}} = \psi^\dagger \bar{\sigma}^\mu \sigma^a \psi). \tag{16}$$

4.3 Encoding the "90-degree orthogonality" in the connection.

To match the 1995 picture (one oscillatory component orthogonal to the other), we choose the $SU(2)$ connection to be *purely off-diagonal* in the (ψ_+, ψ_-) basis:

$$A_\mu = \frac{1}{2}\,a_\mu\,\sigma^2, \tag{17}$$

so that D_μ mixes ψ_+ and ψ_- by a generator that rotates them into each other, representing the locked right-angle coupling. We work in the sheet basis where the mixing generator is σ^2 , that keeps it gauge-clean without changing the equations.

4.4 $SU(2)$ and the Hopf fibration: explicit map

With the normalization Eq: (12), $z(x)$ lives on S^3 and therefore carries a natural Hopf fibration structure

$$S^1 \hookrightarrow S^3 \to S^2. \tag{18}$$

The Hopf (Bloch-sphere) projection is the real unit 3-vector

$$\mathbf{n}(x) \equiv n^a(x)\,\hat{e}_a \quad \text{with} \quad n^a(x) \equiv z^\dagger(x)\,\sigma^a\,z(x), \qquad \mathbf{n}\cdot\mathbf{n} = 1, \tag{19}$$

which provides an explicit $SU(2) \to SO(3)$ geometric picture: $\mathbf{n}(x) \in S^2$ is the *observable polarization/orientation* of the coupled sheets, while the Hopf fiber angle is the *internal sheet-phase*.

The associated $U(1)$ Hopf connection (Berry connection) is

$$a_\mu(x) \equiv -i\, z^\dagger \partial_\mu z, \tag{20}$$

with curvature

$$f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu = \frac{1}{2}\, \epsilon_{abc}\, n^a\, \partial_\mu n^b\, \partial_\nu n^c, \tag{21}$$

i.e. the pullback of the area 2-form on S^2. This is the precise mathematical statement that the coupled two-sheet spinor carries a *twist bundle*: the polarization lives on S^2 while the hidden phase lives on S^1.

Embedding into $SU(2)$. Given $z(x)$, define an $SU(2)$ group element

$$U(x) = \begin{pmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{pmatrix} \in SU(2), \tag{22}$$

so that $U^\dagger \sigma^3 U = n^a \sigma^a$ reproduces (23). This realizes our "rotating sphere of oscillation" as an $SU(2)$ rotor whose base-space projection is the polarization sphere.

We define the normalized spinor $z = \psi/\|\psi\| \in S^3$, whose Hopf projection

$$\mathbf{n} = z^\dagger \vec{\sigma}\, z \in S^2 \tag{23}$$

encodes the observable orientation of the field.

Optics anchor (Stokes/Poincaré sphere). The Hopf projection $\mathbf{n} = z^\dagger \vec{\sigma}\, z \in S^2$ is the standard Poincaré/Bloch representation of polarization: the (normalized) Stokes parameters are

$$S_i \;\propto\; n_i, \qquad i \in \{1,2,3\}, \tag{24}$$

so that the base sphere S^2 in the Hopf fibration is directly identified with the space of observable polarization states, while the Hopf fiber phase corresponds to the unobservable overall spinor phase.

4.5 Antimatter as the opposite sheet

To encode "antimatter lives on the opposite sheet", we implement charge conjugation as a *sheet swap* plus a sign flip of the coupling. Define charge conjugation on the two-sheet spinor as

$$\psi^c \equiv i\sigma^2\psi^* \quad \Rightarrow \quad \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} \mapsto \begin{pmatrix}\psi_-^*\\ -\psi_+^*\end{pmatrix}. \tag{25}$$

The opposite sheet corresponds to the charge-conjugate orientation of the internal spinor bundle (antipodal n), and for neutral excitations this is a geometric dual rather than a distinct particle species Under (25), the Hopf base vector flips:

$$\mathbf{n} \;\mapsto\; -\mathbf{n}, \tag{26}$$

which geometrically means *the state moves to the antipode on* S^2 while the fiber phase reverses orientation. Physically, we interpret this as the same light-geometry viewed on the opposite sheet: the backward-oriented oscillatory component becomes the forward component under conjugation.

To make this dynamical, we assign opposite sign coupling to the ECT torsion vector:

$$\mathcal{L}_{\text{int}} = \frac{\kappa}{2}\,\tau_\mu\,J^\mu_{\text{sheet}} \quad \Rightarrow \quad \mathcal{L}_{\text{int}}(\psi^c) = -\frac{\kappa}{2}\,\tau_\mu\,J^\mu_{\text{sheet}}(\psi), \tag{27}$$

so that antimatter is literally the *oppositely-oriented sheet current* in the same torsion background. This matches the ECT "mirror/opposite phase" interpretation already used when separating envelope from phase structure.

Two-Sheet Hopf Geometry of Light

Forward sheet $\psi_+ \quad \perp \quad$ Backward sheet ψ_-

$SU(2)$ spinor: $\psi = \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} \in S^3$

Hopf map: $\mathbf{n} = z^\dagger\vec{\sigma}z \in S^2$

Antimatter: $\psi \to i\sigma^2\psi^* \;\Rightarrow\; \mathbf{n} \to -\mathbf{n}$

Figure 1: Schematic representation of the two-sheet spinor structure and its Hopf fibration. The orthogonal oscillatory sheets encode forward and backward phase components; charge conjugation maps states to the opposite Hopf sheet.

Antipodal check (opposite sheet map). For a simple reference state take $z = (1, 0)^{\mathsf{T}}$, for which

$$\mathbf{n} = z^{\dagger} \vec{\sigma}\, z = \hat{\mathbf{z}}. \tag{28}$$

Under charge conjugation / opposite-sheet transformation $z \mapsto z^c \equiv i\sigma^2 z^*$,

$$z^c = i\sigma^2 z^* = i\sigma^2 (1, 0)^{\mathsf{T}} = (0, -1)^{\mathsf{T}}, \tag{29}$$

and therefore

$$\mathbf{n}^{\,c} = (z^c)^{\dagger} \vec{\sigma}\, z^c = -\,\hat{\mathbf{z}}. \tag{30}$$

Hence the opposite-sheet map is the antipodal map on the Hopf base: $\mathbf{n} \mapsto -\mathbf{n}$.

4.6 Concrete dynamical test: uniform internal mixing $\Rightarrow$ polarization precession on S^2.

A direct (and simulatable) check of the two-sheet $SU(2)$ structure is obtained by taking a constant internal connection along the propagation direction. Working in the sheet basis where the mixing generator is σ^2, set

$$A_{\mu} = \frac{1}{2}\, a_{\mu}\, \sigma^2, \qquad a_{\mu} = \text{const}, \tag{31}$$

and consider a plane-wave propagation along a coordinate λ (e.g. $\lambda = z$ or $\lambda = t$). Neglecting external sources and any spatial variation other than along λ, the spinor transport reduces to a uniform internal rotation,

$$\frac{d\psi}{d\lambda} = i\, \Omega\, \frac{\sigma^2}{2}\, \psi, \qquad \Omega \equiv g\, a_{\lambda}, \tag{32}$$

so that $\psi(\lambda) = \exp(i\Omega\lambda\, \sigma^2/2)\psi(0)$.

Define the normalized spinor $z = \psi/\|\psi\|$ and the Hopf (Bloch/Poincaré) vector

$$\mathbf{n}(\lambda) = z^{\dagger}(\lambda)\, \vec{\sigma}\, z(\lambda) \in S^2. \tag{33}$$

Using $\frac{d}{d\lambda}(z^{\dagger}\sigma^a z) = z^{\dagger}[i\Omega\sigma^2/2, \sigma^a]z$ and $[\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k$, one obtains the closed precession law

$$\frac{d\mathbf{n}}{d\lambda} = -\,\Omega\, \hat{\mathbf{e}}_2 \times \mathbf{n}, \tag{34}$$

i.e. $\mathbf{n}$ rotates on S^2 about the internal axis $\hat{\mathbf{e}}_2$ at angular rate $|\Omega|$. This is the exact analogue of a Rabi oscillation / polarization rotation: a constant sheet-mixing field produces a predictable precession of the polarization state. Experimentally, this corresponds to an optical rotation (birefringence-like) signature, with accumulated rotation angle

$$\Delta\Phi = \Omega\, \Delta\lambda. \tag{35}$$

(The overall sign depends on the D_μ convention and the choice of generator.)
Equations (34)–(35) provide a minimal dynamical test of the model: given a_λ, the polarization rotation is fixed and can be simulated by integrating (34) or by evolving ψ via (32).

4.7 Polarization precession from uniform sheet mixing

A direct dynamical test of the two-sheet $SU(2)$ formulation is obtained by considering a uniform internal mixing field along the propagation direction. Working in the sheet basis where the mixing generator is σ^2, we take the internal connection to be constant,

$$A_\mu = \frac{1}{2}\, a_\mu\, \sigma^2, \qquad a_\mu = \text{const}, \tag{36}$$

so that the covariant derivative induces a constant internal rotation rate. Restricting attention to propagation along a coordinate λ (e.g. $\lambda = z$ or $\lambda = t$), the spinor transport equation reduces to

$$\frac{d\psi}{d\lambda} = i\, \Omega\, \frac{\sigma^2}{2}\, \psi, \qquad \Omega \equiv g\, a_\lambda, \tag{37}$$

with solution

$$\psi(\lambda) = \exp\!\left(i\, \Omega\, \lambda\, \frac{\sigma^2}{2}\right) \psi(0). \tag{38}$$

Defining the normalized spinor $z = \psi/\|\psi\|$ and the associated Hopf (Bloch/Poincaré) vector

$$\mathbf{n}(\lambda) = z^\dagger(\lambda)\, \vec{\sigma}\, z(\lambda) \in S^2, \tag{39}$$

the evolution of $\mathbf{n}$ follows directly from $\frac{d}{d\lambda}(z^\dagger \sigma^a z) = z^\dagger [\, i\Omega\sigma^2/2, \sigma^a\,] z$ together with the Pauli algebra $[\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k$. One obtains the closed precession equation

$$\frac{d\mathbf{n}}{d\lambda} = -\, \Omega\, \hat{\mathbf{e}}_2 \times \mathbf{n}, \tag{40}$$

where $\hat{\mathbf{e}}_2$ denotes the internal axis associated with σ^2. (The overall sign depends on the convention chosen for the covariant derivative and has no physical effect on the observable rotation rate.)

Equation (40) shows that a constant sheet-mixing field induces a uniform precession of the polarization vector on the Bloch/Poincaré sphere S^2 with angular frequency $|\Omega|$. The accumulated rotation angle over an interval $\Delta\lambda$ is therefore

$$\Delta\Phi = \Omega\, \Delta\lambda. \tag{41}$$

This is the exact analogue of a Rabi oscillation or optical polarization rotation, and corresponds observationally to birefringence or optical activity.

The result provides a minimal but nontrivial dynamical validation of the two-sheet construction: a constant internal $SU(2)$ mixing produces a predictable, gauge-controlled rotation of the observable polarization state. This behaviour can be verified either analytically via (40) or numerically by evolving (38), and constitutes a concrete optical signature of the underlying sheet-coupled spinor dynamics.

4.8 Equations of motion and conserved sheet current

Starting from the spinor–sheet Lagrangian

$$\mathcal{L}_\psi = i\,\psi^\dagger \bar{\sigma}^\mu D_\mu \psi - \mu\,\psi^\dagger \psi + \frac{\kappa}{2}\,\tau_\mu\,\psi^\dagger \bar{\sigma}^\mu \psi, \tag{42}$$

with $D_\mu = \partial_\mu - igA_\mu$ and $\bar{\sigma}^\mu = (\mathbf{1}, -\vec{\sigma})$, variation with respect to $\psi^\dagger$ yields the Weyl equation with sheet-mixing and torsion–current coupling,

$$i\,\bar{\sigma}^\mu D_\mu \psi - \mu\,\psi + \frac{\kappa}{2}\,\tau_\mu\,\bar{\sigma}^\mu \psi = 0. \tag{43}$$

Equivalently, (43) may be written as a Weyl equation with an effective shift of the connection along τ_μ,

$$i\,\bar{\sigma}^\mu \Big(D_\mu - i\,\frac{\kappa}{2}\,\tau_\mu\Big)\psi - \mu\,\psi = 0, \tag{44}$$

making explicit that τ_μ acts as a background "chemical-potential"-like bias for the sheet current.[1]

Noether current. The Lagrangian (42) is invariant under global phase rotations $\psi \mapsto e^{i\alpha}\psi$, which by Noether's theorem yields the conserved sheet current

$$J^\mu_{\text{sheet}} \equiv \psi^\dagger \bar{\sigma}^\mu \psi. \tag{45}$$

Taking the Hermitian conjugate of (43) and combining with (43) in the standard way gives the continuity equation

$$\partial_\mu J^\mu_{\text{sheet}} = 0, \tag{46}$$

provided τ_μ is treated as a fixed background (or, more generally, when τ_μ is dynamical and its equation of motion is sourced consistently by J^μ_{sheet}). In particular, the τ_μ coupling in (42) preserves the global $U(1)$ symmetry and therefore does not spoil current conservation.

Coupled consistency (optional statement). When τ_μ is promoted to a dynamical field, the torsion–phase sector supplies an equation of motion of the schematic form

$$\partial_\nu F^{\nu\mu} + \cdots = \frac{\kappa}{2}\,J^\mu_{\text{sheet}}, \tag{47}$$

[1] The precise sign convention depends on whether the $\tau_\mu J^\mu$ term is taken with $+\kappa/2$ or $-\kappa/2$; the current conservation statement below is unchanged.

so that taking ∂_μ of both sides and using the antisymmetry of $F^{\nu\mu}$ forces $\partial_\mu J^\mu_{\rm sheet} = 0$. Thus current conservation is maintained either as a Noether identity (background τ_μ) or as a compatibility condition of the coupled field equations (dynamical τ_μ).

4.9 Polarization transport, Hopf connection, and geometric phase

Let $z(x) \in \mathbb{C}^2$ be a normalized spinor,

$$z^\dagger z = 1, \tag{48}$$

and define the associated Bloch/Poincaré (Hopf) vector

$$n^a \equiv z^\dagger \sigma^a z, \qquad \mathbf{n} \in S^2, \tag{49}$$

together with the $U(1)$ Hopf (Berry) connection

$$a_\mu \equiv -i\, z^\dagger \partial_\mu z. \tag{50}$$

Under the local phase redundancy $z \mapsto e^{i\alpha(x)} z$, the connection transforms as

$$a_\mu \mapsto a_\mu + \partial_\mu \alpha, \tag{51}$$

so that a_μ is a genuine $U(1)$ gauge potential on the Hopf bundle.

Transport of the polarization vector. Differentiating (49) gives

$$\partial_\mu n^a = (\partial_\mu z^\dagger)\sigma^a z + z^\dagger \sigma^a (\partial_\mu z). \tag{52}$$

It is convenient to separate the unphysical fiber phase by defining the $U(1)$-covariant derivative on the bundle,

$$D_\mu z \equiv (\partial_\mu - i a_\mu)\, z, \qquad z^\dagger D_\mu z = 0, \tag{53}$$

where the orthogonality $z^\dagger D_\mu z = 0$ follows from (48) and the definition (50). In terms of $D_\mu z$, the polarization transport is manifestly gauge-invariant,

$$\partial_\mu n^a = (D_\mu z)^\dagger \sigma^a z + z^\dagger \sigma^a (D_\mu z). \tag{54}$$

Thus only the horizontal (base-space) motion $D_\mu z$ changes the observable polarization $\mathbf{n}$; pure fiber-phase motion shifts a_μ but leaves $\mathbf{n}$ unchanged.

Curvature and the pulled-back area form on S^2. The gauge-invariant curvature (field strength) of the Hopf connection is

$$f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu = -\, i \left(\partial_\mu z^\dagger \partial_\nu z - \partial_\nu z^\dagger \partial_\mu z \right). \tag{55}$$

A standard identity for the Hopf map (49) expresses $f_{\mu\nu}$ purely in terms of the base-space field $\mathbf{n}$ as the pullback of the area two-form on S^2:

$$f_{\mu\nu} = \frac{1}{2}\, \epsilon_{abc}\, n^a\, \partial_\mu n^b\, \partial_\nu n^c. \tag{56}$$

Equation (56) shows that the curvature measures the oriented area swept on the polarization sphere by $\mathbf{n}(x)$.

Berry/Pancharatnam geometric phase (optical signature). For adiabatic transport of the polarization state around a closed loop C in parameter space (or along an optical path), the accumulated geometric phase is

$$\gamma_{\mathrm{geom}}[C] = \oint_C a_\mu\, dx^\mu = \int_{\Sigma(C)} f_{\mu\nu}\, d\Sigma^{\mu\nu}, \tag{57}$$

where $\Sigma(C)$ is any surface spanning C. Using (56), this phase is equal (up to a conventional factor) to the solid angle subtended by the loop traced by $\mathbf{n}$ on S^2. This is precisely the Pancharatnam–Berry phase of polarization optics, providing a clean observable: geometric phase shifts under closed polarization cycles (e.g. polarization loops induced by controlled sheet-mixing or torsion backgrounds).

5 Gravity as Surface Expansion

We now demonstrate how the torsion–phase field reproduces gravitational acceleration in the weak-field regime. The key result is that what is conventionally interpreted as an attractive gravitational force can instead be understood as a kinematic effect arising from gradients of the torsion field associated with shell-wise expansion [1, 2].

5.1 Torsion Potential and Radial Acceleration

Consider a static, spherically symmetric configuration generated by a compact mass M. We take the torsion–phase potential to depend only on the radial coordinate,

$$\tau_\mu = (\tau(r), 0, 0, 0), \tag{58}$$

with $\tau(r)$ a scalar torsion potential.

The physical acceleration experienced by a test body is identified with the gradient of the torsion field,

$$\mathbf{a}(r) = -\nabla \tau(r). \tag{59}$$

For spherical symmetry this reduces to

$$a(r) = -\frac{d\tau}{dr}. \tag{60}$$

Choosing

$$\tau(r) = \frac{GM}{r}, \tag{61}$$

immediately yields

$$a(r) = \frac{GM}{r^2}, \tag{62}$$

which reproduces the Newtonian inverse-square law exactly. Importantly, this result is obtained without introducing a force law; acceleration arises as a geometric response to torsion gradients.

5.2 Interpretation as Surface Expansion

In this framework, local gravity is not interpreted as a downward pull on test masses, but as the upward expansion of nested matter shells driven by torsion tension. A freely falling body follows an inertial trajectory, while the expanding shell geometry accelerates relative to it.

Near the surface of a gravitating body of radius R, the measured gravitational acceleration

$$g = \left.\frac{d^2R}{dt^2}\right|_{r=R} \tag{63}$$

is reinterpreted as the local expansion rate of the outermost compacted shell. Because this expansion is uniform across the shell, all test masses experience identical acceleration, ensuring exact compatibility with the equivalence principle.

This interpretation provides a geometric explanation for why gravitational acceleration is independent of the mass and internal composition of the test body.

The interpretation of gravity and cosmological expansion as dual manifestations of a single shell-based expansion process follows the wavefront formulation developed in [9], where matter formation and large-scale structure arise from resonant ECT shell propagation.

5.3 Metric Correspondence and Weak-Field Limit

Metric correspondence. The relation between the torsion potential and the weak–field metric follows from the Einstein equations sourced by the torsion stress–energy tensor. In the static, weak–field regime the dominant contribution arises from the energy density associated with the torsion configuration, and the 00–component of the field equations reduces to a Poisson-type equation for the Newtonian potential. Since the sourced torsion equation yields $\nabla^2\tau_0 = 4\pi G\rho$ in this regime (Sec. 12), consistency of the coupled system implies that the metric potential and torsion potential coincide up to normalization. Accordingly, in the weak–field limit one may write

$$g_{00} \simeq 1 - \frac{2\tau_0}{c^2}, \tag{64}$$

which reproduces the Schwarzschild form once the exterior solution $\tau_0 = GM/r$ is imposed. In this sense, curvature and torsion gradients encode the same physical information in the weak–field regime, differing only in geometric interpretation. The equivalence between the expansion-based torsion potential used here and the standard weak-field Schwarzschild metric follows directly from the geometric gravity construction developed in [2], of which the present treatment provides a kinematic reinterpretation rather than a modification.

Weak–field derivation. To leading order about Minkowski spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \tag{65}$$

and in Newtonian gauge the metric takes the form

$$ds^2 = -(1 + 2\Phi/c^2)c^2dt^2 + (1 - 2\Phi/c^2)d\vec{x}^2. \tag{66}$$

The 00–component of the linearised Einstein equations gives

$$\nabla^2\Phi = 4\pi G\, T^{(\tau)}_{00}. \tag{67}$$

For a static torsion configuration $\tau_\mu = (\tau_0(\vec{x}), 0, 0, 0)$ with negligible potential term, the torsion stress–energy reduces to

$$T^{(\tau)}_{00} \simeq \frac{1}{2}(\nabla\tau_0)^2, \tag{68}$$

while the sourced torsion field equation yields $\nabla^2\tau_0 = 4\pi G\rho$. Matching the Newtonian limit therefore identifies the metric potential with the torsion potential,

$$\Phi \equiv \tau_0, \tag{69}$$

so that

$$g_{00} \simeq 1 - \frac{2\tau_0}{c^2}. \tag{70}$$

In the weak-field regime the dominant Newtonian source arises from the compaction current sector introduced in Sec. 11, while the torsion energy density governs consistency of the coupled Einstein system. The torsion potential contributes to the spacetime metric through its energy–

momentum tensor. In the weak-field, static limit, the g_{00} component takes the form

$$g_{00} \simeq 1 - \frac{2\tau(r)}{c^2}. \tag{71}$$

Substituting $\tau(r) = GM/r$ gives

$$g_{00} \simeq 1 - \frac{2GM}{c^2 r}, \tag{72}$$

which matches the Schwarzschild solution of General Relativity to leading order.

Thus curvature-based and torsion-gradient descriptions are mathematically equivalent in the weak-field regime, differing only in geometric interpretation. Curvature encodes the same physical information as the torsion-induced expansion field.

Throughout the weak–field analysis we work consistently to leading order in the non–relativistic source density ρ entering through the conserved current J^μ. The torsion self–energy $T^{(\tau)}_{00}$ contributes only at higher order in gradients of τ_0 and does not act as the dominant Newtonian source. The identification $\Phi \simeq \tau_0$ therefore follows from matching solutions of the sourced torsion equation to the linearised Einstein equations at leading order, rather than from a nonlinear self–coupling of the torsion field.

5.4 Free Fall and the Equivalence Principle

In the torsion field picture, free fall corresponds to motion along geodesics of constant torsion phase. Locally inertial frames are those comoving with the expanding shell geometry. As a result:

- All freely falling bodies follow identical trajectories,
- Gravitational acceleration is independent of test mass,
- Local experiments cannot distinguish torsion-induced expansion from inertial motion.

These properties ensure full compatibility with the weak equivalence principle and with all classical tests of gravity in the Newtonian and post-Newtonian regimes.

The reinterpretation of gravity as surface expansion therefore preserves empirical success while offering a unified geometric origin for gravitational acceleration and large-scale expansion phenomena. Before proceeding to observational tests, we now formalise the universal field-level origin of shells, horizons, and discrete scale hierarchies.

6 Universal Field Origin of Shells, Horizons, and Ladders

6.1 Universal Massless Field as Geometric Substrate

All physical structure considered in this work is embedded within a universal massless propagation field characterised by a single invariant causal speed c. Empirically, the electromagnetic field provides the clearest and most thoroughly tested realisation of such a field: it propagates without rest mass, operates across all observed scales, and defines the causal structure of spacetime. In this sense, c is not merely a property of light, but a geometric normalisation linking temporal and spatial intervals.

We therefore take as a foundational assumption that physical reality is organised within a universal expansion geometry governed by massless field propagation at speed c. All subsequent structure—localisation, stability, and hierarchy—must arise internally to this geometry.

6.2 Horizons as Expansion-Limited Wavefronts

Given a finite propagation speed, horizons arise unavoidably. A horizon is defined as the boundary of influence reachable by the universal field within a finite time interval. This definition is independent of gravitational dynamics and applies generically to any causal field.

Examples include particle horizons, event horizons, cosmological horizons, and last-scattering surfaces. In all cases, horizons correspond to outermost expansion wavefronts of the universal field. Within the Expansion–Compaction–Torsion (ECT) framework, horizons are therefore identified as large-scale shells generated purely by expansion dynamics.

6.3 Shell Formation via Phase-Locked Compaction

When the universal expansion field encounters nonlinear feedback—through boundary conditions, self-interaction mediated by matter, or geometric confinement—propagation may partially arrest into standing or quasi-standing configurations. These configurations define shells: phase-locked surfaces embedded within the expansion geometry.

Such shells are observed across physical scales, including atomic orbitals, nuclear energy levels, magnetospheric layers, stellar photospheres, galactic rings, and cosmological surfaces. Their existence indicates that compaction is a generic consequence of wave propagation under confinement.

Within ECT, shell formation corresponds to the Compaction operator acting on the expansion field, producing stable geometric structures without requiring singularities.

6.4 Quantization and the Emergence of Ladders

Stability of closed wave structures imposes a phase-closure condition of the form

$$\oint \nabla\phi \cdot d\ell = 2\pi n, \qquad n \in \mathbb{Z}. \tag{73}$$

This condition is universal across wave physics and leads directly to discrete allowable configurations. As a result, shells do not form continuously but organise into quantised families distinguished by integer winding or mode number.

These discrete families constitute ladders: ordered hierarchies of shells separated by forbidden or unstable intervals. The ladder structure observed in atomic shells, nuclear levels, and large-scale astrophysical systems follows naturally from this quantisation principle.

Torsion plays a central role by permitting twist, chirality, and phase memory within shells, stabilising quantised configurations against collapse. In ECT, ladders arise from the coupled action of Expansion, Compaction, and Torsion.

6.5 Electromagnetic Spectrum as a Projection of the Ladder

The electromagnetic spectrum provides a continuous scale axis characterised by wavelength λ and frequency f, related by

$$c = \lambda f. \tag{74}$$

Empirically, each spectral band couples most strongly to physical structures of comparable characteristic scale. Molecular vibrations, atomic transitions, nuclear processes, magnetospheric oscillations, and cosmological backgrounds all imprint themselves at corresponding wavelengths.

The electromagnetic spectrum therefore does not generate shells or ladders, but acts as a universal probe that projects the multiscale shell hierarchy onto a single invariant carrier field. In this sense, the spectrum constitutes an observational cross-section of the underlying ECT ladder.

6.6 Formal Postulate and Lemma

Definition 2 (Universal Field Postulate)**.** There exists a universal massless propagation field with invariant causal speed c that defines the geometric substrate of spacetime. All physical structures are embedded within this field, and no interaction, signal, or causal influence propagates outside its expansion geometry.

This postulate elevates the empirically established invariance of c from a property of electromagnetism to a foundational geometric principle. The electromagnetic field represents the most directly observable manifestation of this universal expansion field, but the postulate itself is not restricted to a specific field realisation. The universal field postulated here is not identified with any specific gauge field, but represents the underlying causal propagation geometry common to all massless interactions.

Lemma 2 (Shell–Horizon–Ladder Emergence)**.** *Within a universal massless field satisfying Definition 2, the following structures arise generically:*

1. Horizons *emerge as expansion-limited wavefront boundaries determined solely by finite propagation speed.*

2. Shells *arise where expansion undergoes nonlinear feedback or confinement, producing phase-locked standing or quasi-standing configurations.*

3. Ladders *arise because stable shell configurations must satisfy discrete phase-closure conditions of the form*

$$\oint \nabla\phi \cdot d\ell = 2\pi n, \qquad n \in \mathbb{Z}, \tag{75}$$

resulting in quantised families of allowable shell solutions.

Torsion degrees of freedom stabilise these configurations by permitting twist, chirality, and phase memory within shells.

Physical Argument. A finite invariant propagation speed necessarily defines causal boundaries, yielding horizons. Nonlinear feedback or confinement of a propagating field generically produces standing-wave structures, yielding shells. Stability of closed wave structures requires integer phase closure, yielding discrete shell families organised as ladders. These mechanisms are universal features of wave physics and do not depend on the specific identity of the massless field. □

6.7 Unified Interpretation

Shells, horizons, and ladders are not independent phenomena, but distinct manifestations of a single universal field geometry governed by Expansion–Compaction–Torsion dynamics. Horizons correspond to outermost expansion shells, shells correspond to phase-locked compaction surfaces, and ladders correspond to torsion-stabilised quantised hierarchies. The electromagnetic spectrum provides a direct observational projection of this structure across scales.

7 Weak-Field and Solar-System Tests

Any viable modification or reinterpretation of gravity must reproduce the full suite of classical weak-field tests within observational precision. In this section we show that the torsion–expansion framework is empirically indistinguishable from General Relativity in all currently tested Solar-System regimes.

7.1 Newtonian Limit and Orbital Dynamics

For a static, spherically symmetric source with torsion potential

$$\tau(r) = \frac{GM}{r}, \tag{76}$$

This functional form is not assumed a priori, but is recovered explicitly in Sec. 12 from the sourced torsion field equation via Gauss–law integration and matching to the weak–field Einstein equations.

The radial acceleration

$$a(r) = -\frac{d\tau}{dr} = \frac{GM}{r^2} \tag{77}$$

reproduces the Newtonian gravitational field exactly.

As a result, the standard equations governing Keplerian orbits follow unchanged. Circular and elliptical orbits satisfy

$$\frac{v^2}{r} = \frac{GM}{r^2}, \tag{78}$$

yielding the usual relations for orbital periods, energies, and angular momenta. No additional parameters are introduced at this order.

7.2 Perihelion Precession

In the weak-field, slow-motion limit, the metric component

$$g_{00} \simeq 1 - \frac{2GM}{c^2 r} \tag{79}$$

is identical to that of the Schwarzschild solution. Consequently, the relativistic correction to planetary orbits produces the standard perihelion advance

$$\Delta\phi = \frac{6\pi GM}{c^2 a(1 - e^2)}, \tag{80}$$

where a is the semi-major axis and e the eccentricity.

The torsion-based interpretation introduces no deviations at this order, ensuring agreement with Mercury's observed precession and other Solar-System constraints.

7.3 Gravitational Time Dilation

Gravitational time dilation arises from the same metric component g_{00}. The proper time interval $d\tau_p$ measured by a clock at radius r satisfies

$$d\tau_p = \sqrt{g_{00}}\, dt \simeq \left(1 - \frac{GM}{c^2 r}\right) dt. \tag{81}$$

Thus, clocks located deeper in the torsion-gradient field run more slowly, in exact agreement with both gravitational redshift measurements and modern satellite-based timekeeping experiments.

7.4 Light Deflection and Shapiro Delay

Null geodesics are governed by the same weak-field metric. As a result, the deflection angle of light passing a mass M at impact parameter b is

$$\delta\theta = \frac{4GM}{c^2 b}, \tag{82}$$

matching the General Relativistic prediction.

Similarly, the Shapiro time delay for radar signals propagating through the torsion field follows the standard logarithmic form and is observationally indistinguishable from GR within current experimental precision.

7.5 Parameterized Post-Newtonian Consistency

Because the weak-field metric coincides with the Schwarzschild form to leading order, the torsion–expansion framework reproduces the Parameterized Post-Newtonian (PPN) parameters

$$\gamma = 1, \qquad \beta = 1, \tag{83}$$

with all other PPN coefficients vanishing, as in General Relativity.

Therefore, all Solar-System tests sensitive to PPN parameters—including lunar laser ranging, planetary ephemerides, and spacecraft tracking—are satisfied identically.

7.6 Summary of Weak-Field Behaviour

The torsion–expansion theory:

- Reproduces Newtonian gravity exactly,
- Matches General Relativity in the post-Newtonian regime,
- Preserves gravitational time dilation and redshift,
- Predicts standard light deflection and time delay,
- Introduces no new propagating degrees of freedom that modify weak-field gravitational phenomenology.

Around Minkowski backgrounds the theory propagates two tensor modes and up to three torsion-phase modes, which decouple from weak-field gravitational observables. Any observable deviations from General Relativity therefore arise only beyond the weak-field limit, motivating examination of strong-field and cosmological regimes in subsequent sections.

8 Cosmology from Homogeneous Torsion

We now consider the cosmological implications of the torsion–phase field in the covariant field-strength formulation introduced in Sec. 3. The torsion sector is governed by

$$S_\tau = \int d^4x \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(s)\right], \qquad F_{\mu\nu} \equiv \nabla_\mu \tau_\nu - \nabla_\nu \tau_\mu, \qquad s \equiv \tau_\mu \tau^\mu, \tag{84}$$

with field equation

$$\nabla_\mu F^{\mu\nu} - 2V'(s)\tau^\nu = 0. \tag{85}$$

8.1 FRW Background and Homogeneous Ansatz

We adopt a spatially flat Friedmann–Robertson–Walker (FRW) metric,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^{\,2}, \tag{86}$$

where $a(t)$ is the scale factor and $H \equiv \dot{a}/a$ the Hubble parameter.

Homogeneity and isotropy restrict τ_μ to a purely time-like configuration,

$$\bar{\tau}_\mu = (\bar{\tau}_0(t), 0, 0, 0), \qquad \bar{s} = \bar{\tau}_\mu \bar{\tau}^\mu = -\bar{\tau}_0(t)^2. \tag{87}$$

For this homogeneous time-like ansatz, all components of the field strength vanish:

$$\bar{F}_{0i} = \partial_0 \bar{\tau}_i - \partial_i \bar{\tau}_0 = 0, \qquad \bar{F}_{ij} = 0, \qquad \Rightarrow \qquad \bar{F}_{\mu\nu} = 0. \tag{88}$$

8.2 Background Field Condition

Substituting $\bar{F}_{\mu\nu} = 0$ into the torsion equation of motion Eq. (85) yields the background constraint

$$-2V'(\bar{s})\,\bar{\tau}^\nu = 0 \qquad \Rightarrow \qquad V'(\bar{s}) = 0 \quad \text{or} \quad \bar{\tau}^\nu = 0. \tag{89}$$

Therefore, a nontrivial homogeneous torsion background is supported when the field sits at an extremum of the potential, $\bar{s} = \bar{s}_0$ with $V'(\bar{s}_0) = 0$. In this regime the background torsion is "phase-locked" in the sense that it remains stationary in the absence of spatial gradients.

8.3 Effective Energy Density, Pressure, and Equation of State

The torsion stress–energy tensor for the field-strength theory is

$$T^{(\tau)}_{\mu\nu} = F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} + 2V'(s)\tau_\mu \tau_\nu - g_{\mu\nu} V(s). \tag{90}$$

On the homogeneous background, $\bar{F}_{\mu\nu} = 0$ and $V'(\bar{s}_0) = 0$, so the stress–energy reduces to

$$\bar{T}^{(\tau)}_{\mu\nu} = -g_{\mu\nu} V(\bar{s}_0). \tag{91}$$

It follows immediately that the torsion background behaves as an effective cosmological constant, with

$$\rho_\tau = V(\bar{s}_0), \qquad p_\tau = -V(\bar{s}_0), \qquad w_\tau \equiv \frac{p_\tau}{\rho_\tau} = -1. \tag{92}$$

Thus the theory admits a stable accelerating FRW background without introducing vacuum-energy tuning or additional scalar degrees of freedom: the accelerating component arises from the geometric torsion potential evaluated at a phase-locked extremum.

8.4 Friedmann Equations

Including ordinary matter and radiation with energy density ρ_m and pressure p_m, the Einstein equations give

$$3H^2 = 8\pi G \left(\rho_m + V(\bar{s}_0)\right), \tag{93}$$

$$-2\dot{H} = 8\pi G \left(\rho_m + p_m\right). \tag{94}$$

The torsion sector contributes only through the constant term $V(\bar{s}_0)$ at the background level, producing late-time acceleration when it dominates over matter.

8.5 Stability of the Homogeneous Background

Perturbative stability of the torsion background is controlled by the curvature of the potential at the extremum. Writing $\bar{s} = \bar{s}_0 + \delta s$ with $V'(\bar{s}_0) = 0$, stability requires

$$V''(\bar{s}_0) > 0, \tag{95}$$

ensuring that small fluctuations do not drive runaway behaviour of the torsion magnitude. A full SVT perturbation analysis about FRW is deferred to Sec. 10 and Appendix A. A complete SVT decomposition and the reduced quadratic actions (ghost/speed/mass conditions) are presented in Appendix D. The existence of a stable phase-locked vacuum with a finite curvature of the torsion potential, and hence a gapped fluctuation spectrum, parallels the geometric mass-gap mechanism derived for non-Abelian gauge fields in the ECT formulation of Yang–Mills theory [10].

8.6 Unified Interpretation

Within the torsion–expansion framework, the same field admits two complementary regimes:

- *Local gravity:* spatial torsion gradients (nonzero $F_{\mu\nu}$) generate effective accelerations and reproduce the Newtonian limit.
- *Cosmic acceleration:* a homogeneous phase-locked torsion background ($F_{\mu\nu} = 0$, $V'(\bar{s}_0) = 0$) generates an effective cosmological constant with $w_\tau = -1$.

This provides a unified geometric origin for local gravitational acceleration and late-time cosmic expansion within a single covariant torsion field theory.

9 Strong Fields

We now examine the behaviour of the torsion–expansion framework in the strong-field regime. While the weak-field limit reproduces General Relativity exactly, strong gravitational fields probe the nonlinear structure of the torsion field and its coupling to spacetime curvature.

9.1 Phenomenological strong-field parameterisation

The purpose of this section is not to present an exact strong-field solution of the full coupled Einstein–torsion system, but to introduce a controlled phenomenological parameterisation of possible strong-field deviations consistent with the weak-field limit. While the Newtonian and post-Newtonian regimes of the theory are derived directly from the sourced torsion field equations, explicit analytic solutions of the fully nonlinear system remain to be constructed. Accordingly, we model the leading strong-field behaviour through an effective expansion of the exterior torsion profile, which captures the lowest-order departures from the Schwarzschild geometry while preserving all tested weak-field properties.

Outside a compact, spherically symmetric mass distribution, we therefore consider a static torsion configuration depending only on the radial coordinate. Motivated by the weak-field solution and dimensional analysis, we parameterise the exterior torsion potential as

$$\tau(r) = \frac{GM}{r} + \frac{\beta}{r^2}, \tag{96}$$

where the first term is the unique exterior weak-field solution derived in Sec. 12, and the coefficient β encodes phenomenological strong-field corrections arising from nonlinear torsion dynamics and shell compaction effects. The parameter β is not fixed a priori by the present analysis and serves to quantify possible deviations from Schwarzschild behaviour in the strong-field regime.

The effective gravitational potential is identified as

$$\Phi(r) \equiv -\tau(r). \tag{97}$$

9.2 Effective Metric and Horizon Structure

In the static, spherically symmetric case, the spacetime metric may be written in Schwarzschild-like form,

$$ds^2 = -f(r)c^2dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{98}$$

with

$$f(r) = 1 - \frac{2GM}{c^2 r} - \frac{2\beta}{c^2 r^2}. \tag{99}$$

In the limit $\beta \to 0$, this reduces exactly to the Schwarzschild solution of General Relativity. For nonzero β, the $1/r^2$ correction represents the leading strong-field torsion contribution.

The event horizon is located at the largest root of $f(r) = 0$, satisfying

$$r_h^2 - \frac{2GM}{c^2} r_h - \frac{2\beta}{c^2} = 0, \tag{100}$$

with solution

$$r_h = \frac{GM}{c^2}\left[1 + \sqrt{1 + \frac{8\beta c^2}{(2GM)^2}}\right]. \tag{101}$$

Thus torsion modifies the horizon radius by an amount controlled by β. For astrophysical black holes, observational constraints require $|\beta| \ll (GM)^2/c^2$, ensuring that deviations from the Schwarzschild radius remain small. We emphasise that this form represents a phenomenological effective description of the exterior geometry. Derivation of exact strong-field solutions of the coupled field equations is deferred to future work.

9.3 Physical Interpretation

Within the torsion–expansion framework, black holes correspond to regions where torsion gradients become sufficiently strong that shell-wise expansion is fully phase-locked. The event horizon marks the surface at which outward expansion relative to external observers ceases, rather than a singular geometric breakdown.

This interpretation does not alter the causal structure of spacetime outside the horizon, nor does it affect observable properties such as orbital motion, light deflection, or redshift at radii $r \gg r_h$.

9.4 Compatibility with Strong-Field Tests

Because the exterior metric differs from Schwarzschild only at order $1/r^2$, all currently tested strong-field phenomena remain consistent with observations. In particular:

- Stellar orbits around Sagittarius A* constrain deviations to be small,
- Black-hole shadow sizes measured by the Event Horizon Telescope are consistent with $\beta \approx 0$,
- Gravitational-wave ringdown frequencies are dominated by the Schwarzschild term.

The torsion–expansion theory therefore passes all existing strong-field tests provided the torsion correction remains subdominant, while offering a controlled parameter through which future deviations may be explored.

9.5 Outlook for Strong-Field Phenomena

Potential observational signatures of torsion effects in the strong-field regime include small shifts in quasinormal mode spectra, modified near-horizon lensing, and subtle deviations in black-hole shadow profiles. These effects lie near the threshold of current experimental sensitivity and provide natural targets for future tests.

A detailed perturbative analysis of strong-field torsion dynamics is deferred to future work.

10 Linear Perturbations and Stability

A consistent gravitational field theory must possess a well-posed perturbative structure: small fluctuations about physically relevant backgrounds should propagate stably, without ghost degrees of freedom, tachyonic instabilities, or ill-posed dynamics. In this section we outline the linear perturbation framework of the torsion–expansion theory and establish the conditions under which the model is perturbatively well behaved.

We consider perturbations about a background solution $(\bar{g}_{\mu\nu}, \bar{\tau}_\mu)$ of the field equations,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \qquad \tau_\mu = \bar{\tau}_\mu + \delta\tau_\mu, \tag{102}$$

and expand the action to quadratic order in $(h_{\mu\nu}, \delta\tau_\mu)$. The resulting second-order action governs the linearised dynamics, stability, and propagating degrees of freedom of the theory.

10.1 Minkowski Background and Local Stability

We first consider perturbations about flat spacetime with a constant torsion background,

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu}, \qquad \bar{\tau}_\mu = \tau_\mu^{(0)} = \text{const.} \tag{103}$$

Expanding the torsion action to quadratic order in $\delta\tau_\mu$ yields

$$S_\tau^{(2)} = \frac{1}{2}\int d^4x \left[\partial_\mu \delta\tau_\nu \partial^\mu \delta\tau^\nu - M^2_{\mu\nu}\, \delta\tau^\mu \delta\tau^\nu\right], \tag{104}$$

where the effective mass matrix is

$$M^2_{\mu\nu} = 2V'(s_0)\eta_{\mu\nu} + 4V''(s_0)\bar{\tau}_\mu\bar{\tau}_\nu, \qquad s_0 = \bar{\tau}_\mu\bar{\tau}^\mu. \tag{105}$$

Local stability of the torsion sector therefore requires

$$V'(s_0) > 0, \qquad V'(s_0) + 2s_0V''(s_0) > 0, \tag{106}$$

ensuring positive-definite kinetic and mass terms and the absence of ghost or tachyonic instabilities. Under these conditions, the torsion field propagates healthy transverse and longitudinal modes with well-posed hyperbolic equations of motion.

Because the metric perturbations retain the Einstein–Hilbert quadratic structure, the spin–2 sector propagates exactly the two transverse graviton polarizations of General Relativity, and the weak-field gravitational phenomenology remains unaltered. A complete Minkowski-space mode decomposition and dispersion-spectrum derivation is provided in Appendix C.

10.2 Cosmological Perturbations

We next consider linear perturbations about the homogeneous and isotropic FRW background,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^{\,2}, \qquad \bar{\tau}_\mu = (\bar{\tau}_0, 0, 0, 0), \tag{107}$$

for which the background field strength vanishes, $\bar{F}_{\mu\nu} = 0$.

Perturbations of the torsion field are decomposed in standard scalar–vector form,

$$\delta\tau_\mu = (\delta\tau_0, \partial_i \delta\tau_L + \delta\tau_i^{(V)}), \qquad \partial^i \delta\tau_i^{(V)} = 0, \tag{108}$$

with metric perturbations decomposed analogously in scalar–vector–tensor (SVT) form.

Scalar sector

For a purely time-like homogeneous background, the scalar perturbations $\delta\tau_0$ and $\delta\tau_L$ enter the quadratic action without time derivatives and act as constraint fields. After integrating out non-dynamical constraint variables, the scalar sector is significantly restricted. In general, at most one longitudinal torsion scalar mode remains, whose propagation is controlled by the curvature of the potential about the background configuration. In the phase–locked late–time regime relevant for cosmology, this mode is either non–propagating or acquires a large effective mass set by $V''(\bar{s})$, leaving no light additional scalar degrees of freedom that modify late–time gravitational phenomenology. This behaviour is made explicit by the reduced scalar quadratic action derived in Appendix D.

As a result, the scalar sector is free of ghost and gradient instabilities at linear order, and the homogeneous torsion background does not introduce new dynamical scalar modes.

Vector sector

The effective mass scales of the torsion perturbations are most transparently defined from the full quadratic expansion of the action about a homogeneous background, as derived in Appendices C and D. In this formulation, the transverse (vector) torsion modes acquire an effective mass

$$m_V^2 \equiv 2V'(\bar{s}), \tag{109}$$

while the longitudinal (scalar) torsion mode is governed by

$$m_L^2 \equiv 2V'(\bar{s}) + 4\bar{s}\, V''(\bar{s}), \tag{110}$$

where $\bar{s} = \bar{\tau}_\mu \bar{\tau}^\mu$. These expressions follow directly from the eigenvalues of the quadratic mass matrix and apply in both Minkowski and FRW backgrounds. In particular, on a phase–locked background satisfying $V'(\bar{s}) = 0$, the transverse modes are massless, $m_V^2 = 0$, while the longitudinal mode is controlled solely by the curvature of the potential through $V''(\bar{s})$.

Tensor sector

Tensor perturbations of the metric propagate exactly as in General Relativity. Gravitational waves travel at the speed of light and experience no leading-order modification in either their dispersion relation or polarization structure.

Summary

For a timelike phase–locked background $\bar{s} = -\bar{\tau}_0^2 < 0$ satisfying $V'(\bar{s}) = 0$, the longitudinal (scalar) torsion mode has $m_S^2 = 4\bar{s}\,V''(\bar{s})$ (Appendix D) and is tachyon-free provided $m_S^2 \geq 0$, i.e. $\bar{s}\,V''(\bar{s}) \geq 0$, which for $\bar{s} < 0$ corresponds to $V''(\bar{s}) \leq 0$.

10.3 Interpretation of Perturbation Modes

The linearised structure of the theory admits a clear physical interpretation:

Tensor modes correspond to curvature waves identical to those of General Relativity. Vector and longitudinal modes of $\delta\tau_\mu$ correspond to torsion-phase fluctuations. Scalar mixtures represent local expansion–compaction phase drift about the background configuration.

Stability of these modes is guaranteed by the existence of a phase-locked minimum of the torsion potential, characterised by $V'(s_0) = 0$ and $V''(s_0) > 0$, ensuring bounded energy density and controlled propagation. In the particle-physics sector, analogous torsion–phase drift mechanisms give rise to neutrino flavour oscillations and CP asymmetry through relative phase evolution between nested torsion shells, as developed quantitatively in [11].

10.4 Strong-Field Perturbations and Observables

In strong-field backgrounds, such as static spherically symmetric compact objects, perturbations may be decomposed into spherical harmonics and separated into odd- and even-parity sectors. Linearisation about the background $(\bar{g}_{\mu\nu}(r), \bar{\tau}_\mu(r))$ yields coupled wave equations governing quasinormal ringing, near-horizon scattering, and lensing corrections.

The leading observational effects of torsion dynamics arise as perturbative shifts in:

- quasinormal mode frequencies,
- black-hole shadow profiles,
- near-horizon photon trajectories.

[1]Earlier heuristic expressions for the effective mass are recovered as special limits of these general formulas when expanding about a purely timelike background.

These quantities depend on the background torsion profile and provide direct observational channels through which the theory may be constrained. A detailed mode-by-mode perturbation analysis is deferred to future work.

10.5 Summary of Perturbative Consistency

The torsion–expansion framework admits a well-posed perturbative formulation. Around physically relevant backgrounds:

- the graviton sector remains identical to General Relativity,
- torsion modes propagate dynamically but remain stable for bounded potentials,
- cosmological backgrounds possess stable accelerating attractors,
- observational deviations are confined to strong-field or precision regimes.

These results establish the perturbative viability of the theory and justify the interpretation of torsion dynamics as a controlled extension of gravitational geometry.

11 Operator Identification and Source Coupling

In the preceding sections the torsion–phase field τ_μ was introduced as a covariant dynamical quantity governing the interaction between expansion and compaction. We now make this identification more precise by relating τ_μ to the torsion operator of the Expansion–Compaction–Torsion (ECT) framework and by introducing its minimal coupling to matter.

Within ECT, torsion represents phase misalignment between the expansive and compactive geometric tendencies. At the field-theoretic level, this misalignment is encoded by a covariant potential whose gradients measure local phase tension. We therefore identify τ_μ as the effective gauge potential associated with the torsion operator $\mathbf{T}$,

$$\tau_\mu \equiv \langle \Psi \,|\, \mathbf{T}_\mu \,|\, \Psi \rangle, \tag{111}$$

where $|\Psi\rangle$ denotes the local geometric state of the expansion–compaction system. This identification does not modify the geometric structure of spacetime, which remains Levi–Civita, but promotes torsion-phase to a genuine dynamical degree of freedom whose stress–energy contributes to gravitational dynamics. The identification of τ_μ as a gauge-like geometric potential and its coupling to an effective source current parallels the torsion–gauge formulation developed for quantum electrodynamic and chromodynamic sectors in [8]. Also the identification of τ_μ as the expectation value of the torsion operator τ_μ relies on the closure and representation properties of the ECT operator algebra, as formalised in the minimal three–generator Lie framework of [3].

In this formulation, matter is interpreted as a localized compaction of geometric shells. The natural source for torsion-phase is therefore a compaction current constructed from the matter sector. To lowest order, we introduce the minimal interaction

$$S_{\text{int}} = \int d^4x\sqrt{-g}\,\tau_\mu J^\mu, \tag{112}$$

where J^μ is a covariant compaction current derived from the matter stress–energy tensor. In the simplest case relevant to non-relativistic sources,

$$J^\mu \approx \kappa\,\rho\,u^\mu, \tag{113}$$

with ρ the rest-mass density, u^μ the matter four-velocity, and κ a coupling constant fixed by the Newtonian limit.

Status of the compaction current. The current J^μ represents the effective macroscopic source of the torsion–phase field, encoding the localisation and compaction of matter. At the present stage, J^μ is introduced as an effective phenomenological current, in direct analogy with charge–current densities in gauge field theories. It is not taken to represent a new independent fundamental field, but an emergent quantity expected to arise from appropriate contractions or projections of the matter stress–energy tensor in a more complete microphysical formulation. In the broader Expansion–Compaction–Torsion (ECT) framework motivating this work, one expects J^μ to admit a first–principles derivation from the matter sector through geometric shell compaction and phase–coherence operators, relating localisation of mass–energy to torsion–phase currents. Development of this microphysical construction, and its explicit relation to $T_{\mu\nu}$, will be presented in future work.

Consistency of the sourced torsion equation implies covariant current conservation. Taking the covariant divergence of the field equation and using the antisymmetry structure inherited from $F_{\mu\nu}$ yields the identity

$$\nabla_\mu J^\mu = 0, \tag{114}$$

so that J^μ is automatically conserved. In this work, we restrict attention to the effective exterior form of J^μ, sufficient to derive the Newtonian and weak–field limits, while a first-principles derivation from the matter sector is deferred to future work.

The torsion field equation is thereby promoted to the sourced form

$$\nabla_\rho\nabla^\rho\tau_\nu + R^\mu{}_\nu\tau_\mu - 2V'(s)\tau_\nu = J_\nu. \tag{115}$$

In writing Eq. (115) we have fixed a Lorenz–type gauge (or, equivalently, integrated out the non–dynamical constraint enforcing it), so that the term $-\nabla_\nu(\nabla_\mu\tau^\mu)$ does not contribute at leading order.

11.1 Weak-Field and Newtonian Limit

To establish the emergence of Newtonian gravity, we consider the weak-field, static, non-relativistic regime,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \tau_\mu = (\tau_0(\vec{x}), 0, 0, 0), \qquad J^\mu = (\kappa\rho, 0, 0, 0),$$

with $|h_{\mu\nu}| \ll 1$ and time derivatives negligible.

To leading order, the torsion field equation reduces to

$$\nabla^2 \tau_0 = \kappa\rho, \tag{116}$$

which is a Poisson equation for the torsion potential. For a compact source of total mass

$$M = \int \rho \, d^3x,$$

the exterior solution is

$$\tau_0(r) = \frac{\kappa M}{4\pi r}. \tag{117}$$

Identifying $\kappa = 4\pi G$ yields

$$\tau_0(r) = \frac{GM}{r}, \tag{118}$$

so that the Newtonian gravitational potential arises dynamically from the sourced torsion field. The physical acceleration of a test body,

$$\vec{a} = -\vec{\nabla}\tau_0, \tag{119}$$

therefore reproduces the inverse-square law without the introduction of a fundamental force.

In this way, the torsion–expansion framework derives the Newtonian limit from a sourced geometric field equation rather than imposing it as an ansatz. Gravitational mass appears as compaction charge, sourcing torsion-phase gradients in the surrounding geometric medium.

11.2 ECT Interpretation

Within the ECT framework, the sourced torsion field represents the geometric response of expansion to localized compaction. The Poisson limit corresponds to the lowest-order balance condition between expansion and compaction operators, while nonlinear and threshold corrections encoded in the torsion potential $V(s)$ govern departures from Newtonian behaviour in strong-field and cosmological regimes.

The identification of τ_μ as the effective potential of the torsion operator provides a direct bridge between the operator hierarchy of ECT and the covariant field-theoretic description developed here. Local gravitational acceleration, cosmological expansion, and strong-field torsion effects are thereby unified as different dynamical regimes of a single sourced geometric phase field.

12 Derived Weak-Field Poisson Limit

A central requirement of any covariant gravitational theory is that it reproduce the Newtonian limit for weak, slowly varying fields sourced by non-relativistic matter. In this section we derive the Poisson limit of the torsion–phase field directly from the sourced field equation and show that the familiar inverse-square acceleration arises as the unique exterior solution for isolated masses.

12.1 Weak-Field Assumptions and Gauge Choice

We work in the weak-field, slow-motion regime about Minkowski spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \tag{120}$$

and consider a static configuration sourced by non-relativistic matter with rest-mass density $\rho(\vec{x})$ and four-velocity $u^\mu \simeq (1,0,0,0)$. Time derivatives are neglected to leading order,

$$\partial_t h_{\mu\nu} \approx 0, \qquad \partial_t \tau_\mu \approx 0. \tag{121}$$

We take the torsion–phase field to be dominated by a time-like potential component,

$$\tau_\mu \simeq \big(\tau_0(\vec{x}), 0, 0, 0\big), \tag{122}$$

consistent with static spherical sources and with the identification of τ_0 as the effective gravitational potential in the torsion-gradient picture.

12.2 Sourced Torsion Equation and Poisson Reduction

Including the minimal compaction-current coupling introduced previously,

$$S_{\rm int} = \int d^4x \sqrt{-g}\, \tau_\mu J^\mu, \tag{123}$$

the torsion field equation takes the sourced form

$$\nabla_\rho \nabla^\rho \tau_\nu + R^\mu{}_\nu \tau_\mu - 2V'(s)\tau_\nu = J_\nu, \qquad s \equiv \tau_\mu \tau^\mu. \tag{124}$$

In the weak-field regime, curvature corrections are higher order,

$$R^\mu{}_\nu \tau_\mu = \mathcal{O}(h\, \partial^2 \tau), \tag{125}$$

and may be neglected at leading Newtonian order. Furthermore, we assume that the torsion potential admits a low-field regime in which the effective mass is negligible,

$$V'(s) \approx 0 \qquad \text{for } |s| \ll s_\star, \tag{126}$$

so that the torsion field behaves as a massless sourced potential at large radii. Under these assumptions, Eq. (124) reduces to

$$\nabla_\rho \nabla^\rho \tau_\nu \simeq J_\nu. \tag{127}$$

For static fields, the covariant d'Alembertian reduces to the spatial Laplacian on the dominant component,

$$\nabla_\rho \nabla^\rho \tau_0 \simeq \nabla^2 \tau_0, \tag{128}$$

and with $J^\mu \simeq (\kappa\rho, 0, 0, 0)$ we obtain the Poisson equation

$$\nabla^2 \tau_0 = \kappa\, \rho. \tag{129}$$

Here ρ should be understood as an effective compaction density rather than the full relativistic energy density.

12.3 Exterior Solution and Identification of the Coupling

For an isolated compact source of total mass

$$M = \int \rho(\vec{x})\, d^3x, \tag{130}$$

the exterior region satisfies $\rho = 0$, and Eq. (129) reduces to Laplace's equation,

$$\nabla^2 \tau_0 = 0 \qquad (r > R_{\rm src}). \tag{131}$$

Assuming spherical symmetry, the unique solution that is regular at spatial infinity is

$$\tau_0(r) = \frac{A}{r}, \tag{132}$$

with constant A fixed by Gauss' law. Integrating Eq. (129) over a sphere enclosing the source yields

$$\int \nabla^2 \tau_0\, d^3x = \oint \nabla \tau_0 \cdot d\vec{S} = \kappa \int \rho\, d^3x = \kappa M. \tag{133}$$

For $\tau_0 = A/r$, we have $\nabla \tau_0 = -A\, \hat{r}/r^2$ and therefore

$$\oint \nabla \tau_0 \cdot d\vec{S} = -4\pi A, \tag{134}$$

so that

$$A = -\frac{\kappa M}{4\pi}. \tag{135}$$

Defining the torsion potential with the conventional attractive sign, we write

$$\tau_0(r) = \frac{\kappa M}{4\pi r}. \tag{136}$$

Matching to the Newtonian potential fixes the coupling

$$\kappa = 4\pi G, \tag{137}$$

and thus

$$\tau_0(r) = \frac{GM}{r}. \tag{138}$$

12.4 Acceleration and Metric Correspondence

In the torsion-gradient interpretation, the physical acceleration of a test body is identified as

$$\vec{a} = -\vec{\nabla}\tau_0. \tag{139}$$

Using Eq. (138) yields

$$a(r) = \frac{GM}{r^2}, \tag{140}$$

recovering the Newtonian inverse-square law as a derived consequence of the sourced torsion–phase field equation.

Moreover, in the weak-field static limit the metric component g_{00} may be written as

$$g_{00} \simeq 1 - \frac{2\tau_0}{c^2}, \tag{141}$$

so that Eq. (138) reproduces the standard Schwarzschild weak-field expansion,

$$g_{00} \simeq 1 - \frac{2GM}{c^2 r}. \tag{142}$$

12.5 Validity Domain and Yukawa Corrections

If the low-field regime (126) is relaxed, the potential term generates an effective mass scale

$$m_{\text{eff}}^2 \equiv 2V'(s_0), \tag{143}$$

and Eq. (129) is replaced by a Yukawa-type equation,

$$\nabla^2\tau_0 - m_{\text{eff}}^2\tau_0 = \kappa\rho. \tag{144}$$

The exterior solution becomes

$$\tau_0(r) = \frac{GM}{r} e^{-m_{\text{eff}} r}, \tag{145}$$

which is tightly constrained by Solar-System tests. Consistency with observations therefore implies that $m_{\text{eff}} r \ll 1$ across the regimes where General Relativity has been confirmed, while allowing the potential $V(s)$ to produce controlled deviations in strong-field or cosmological settings.

This completes the derivation of the Newtonian limit from the sourced torsion–phase field dynamics.

13 Predictions and Falsifiability

The torsion–expansion framework is designed to reproduce General Relativity in all regimes where it has been experimentally verified. Consequently, any observational distinction between the two theories must arise either in strong-field environments or in precision cosmological measurements. This section summarises the principal testable predictions of the theory.

13.1 Strong-Field Signatures

The presence of the torsion parameter β in the strong-field metric introduces controlled deviations from the Schwarzschild solution at order $1/r^2$. Observable consequences include:

- Small shifts in black-hole shadow diameters relative to General Relativity,
- Perturbative corrections to quasinormal mode frequencies in gravitational-wave ringdown,
- Modified near-horizon lensing behaviour for photons grazing compact objects.

Current observational bounds require these effects to be small, but improving angular resolution and gravitational-wave sensitivity may render them testable in the near future.

13.2 Precision Timekeeping and Clocks

Because torsion contributes to the effective gravitational potential, ultra-precise clock comparisons performed at differing gravitational potentials may detect higher-order deviations from standard redshift predictions. Such tests provide a clean laboratory for probing torsion gradients independent of astrophysical modelling uncertainties.

13.3 Cosmological Evolution

At cosmological scales, the theory predicts that dark-energy-like behaviour emerges dynamically from a homogeneous torsion background. Deviations from ΛCDM may arise through:

- Time evolution of the effective equation-of-state parameter w_τ,
- Coupling between torsion dynamics and curvature during early or late-time expansion,
- Subtle effects on structure growth and expansion history.

These effects can be constrained through precision cosmological observations without invoking additional scalar fields or vacuum-energy tuning.

13.4 Falsification Criteria

The torsion–expansion theory would be falsified by any of the following observations:

- Detection of weak-field deviations inconsistent with the Schwarzschild metric,
- Strong-field measurements ruling out all $1/r^2$ corrections at astrophysical scales,
- Cosmological data conclusively excluding any dynamical dark-energy component.
- Neutrino oscillation patterns and CP-violating phase evolution provide an independent observational channel for testing torsion–phase dynamics, as predicted by the shell-phase drift mechanism developed in [11].

Thus the theory is predictive, testable, and empirically constrained.

14 Conclusion

We have presented a covariant torsion field theory in which gravity and cosmological expansion emerge from a single geometric mechanism. By promoting torsion to a dynamical field encoded through a torsion–phase potential, gravitational acceleration arises from spatial torsion gradients, while homogeneous torsion dynamics generate an effective dark-energy sector.

The theory reproduces all tested predictions of General Relativity in the weak-field and Solar-System regimes, remains consistent with current strong-field observations, and naturally extends to cosmology without introducing additional exotic degrees of freedom. The reinterpretation of gravity as shell-wise expansion provides a unified geometric picture linking local and global dynamics.

Crucially, the framework is minimal, covariant, and falsifiable. Any observable deviation from General Relativity is confined to regimes where existing theories already face conceptual or empirical tension.

This work establishes the foundational field-theoretic structure of the torsion–expansion framework. Extensions to quantum fields, algebraic structures, and detailed observational analyses are left to companion studies. Taken together, the results of this paper should be viewed as a refinement of the geometric gravity model introduced in [2], providing a more direct physical interpretation of gravitational acceleration while preserving full agreement with observational tests.

15 Appendix

A Perturbation Roadmap and Programme of Analysis

This appendix outlines the perturbative programme required to fully characterise the dynamical, stability, and observational properties of the torsion–expansion field theory. The purpose of this roadmap is to make explicit the sequence of calculations needed to establish the theory as a complete gravitational framework and to connect its covariant structure to measurable predictions.

A1 Quadratic Action and Degree-of-Freedom Count

The starting point for all perturbative analyses is the second-order expansion of the total action about a background solution $(\bar{g}_{\mu\nu}, \bar{\tau}_\mu)$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \qquad \tau_\mu = \bar{\tau}_\mu + \delta\tau_\mu, \tag{146}$$

with the action expanded to $\mathcal{O}(h^2, \delta\tau^2, h\,\delta\tau)$.

From the quadratic action, one must:

- identify constraint variables,
- integrate out non-dynamical fields,
- construct the reduced kinetic matrix for propagating modes,
- count physical degrees of freedom.

This establishes the precise field content of the theory and ensures the absence of hidden ghost modes.

A2 Local and Minkowski-Space Perturbations

Perturbations about Minkowski or weakly curved backgrounds provide the most direct test of internal consistency. The programme consists of:

- derivation of linearised field equations,
- decomposition into transverse, longitudinal, and scalar sectors,
- computation of dispersion relations,
- identification of propagation speeds and effective masses,
- verification of positive-definite kinetic terms.

This stage determines the behaviour of torsion-phase waves, establishes hyperbolicity of the equations, and constrains the admissible forms of the torsion potential $V(s)$.

A3 Cosmological Perturbations

On an FRW background, the full scalar–vector–tensor (SVT) decomposition is required. The perturbative programme consists of:

- construction of the quadratic action in Fourier space,
- elimination of non-dynamical lapse and shift variables,
- diagonalisation of the scalar sector,
- computation of the kinetic and gradient matrices,
- extraction of sound speeds and effective masses.

From this, one obtains:

- no-ghost and no-gradient-instability conditions,
- evolution equations for structure growth,
- modifications to gravitational slip and lensing,
- predictions for late-time cosmic acceleration.

This analysis determines whether the homogeneous torsion background represents a stable accelerating attractor and how torsion dynamics modify cosmological observables.

A4 Strong-Field and Black-Hole Perturbations

For static spherically symmetric backgrounds, perturbations are to be decomposed into tensorial spherical harmonics and separated into odd- and even-parity sectors. The programme consists of:

- derivation of coupled master equations,
- identification of effective potentials,
- computation of quasinormal mode spectra,
- analysis of near-horizon scattering and stability,
- calculation of lensing and shadow corrections.

These results directly determine the observational signatures of torsion dynamics in black-hole and compact-object environments.

A5 Observational Interface

The perturbative framework provides a direct mapping from theoretical parameters to measurable quantities. In particular:

- weak-field perturbations constrain fifth-force and wave-propagation effects,
- cosmological perturbations constrain background evolution and growth history,
- strong-field perturbations constrain quasinormal ringing and horizon-scale geometry.

Together, these analyses define the falsifiable domain of the torsion–expansion theory and specify the experimental channels through which it may be tested.

A6 Relation to the ECT Framework

Within the broader Expansion–Compaction–Torsion programme, the perturbative structure described above provides the operational realisation of phase-locking, shell stability, and torsion thresholds. Stable backgrounds correspond to phase-coherent fixed points, while propagating torsion modes represent controlled deviations from expansion–compaction balance. Strong-field and cosmological departures from General Relativity arise as nonlinear and threshold regimes of the same underlying torsion-phase dynamics.

This roadmap establishes the sequence of analyses required to elevate the present work from a foundational field-theoretic construction to a fully predictive gravitational framework.

B Variation of the torsion stress–energy tensor

We derive here the stress–energy tensor associated with the torsion–phase sector,

$$S_\tau = \int d^4x\,\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(s)\right], \qquad F_{\mu\nu} \equiv \nabla_\mu\tau_\nu - \nabla_\nu\tau_\mu, \qquad s \equiv \tau_\mu\tau^\mu. \tag{147}$$

The stress–energy tensor is defined by

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_\tau}{\delta g^{\mu\nu}}. \tag{148}$$

B.1 Independence of $F_{\mu\nu}$ from the connection

Using the Levi–Civita connection,

$$\nabla_\mu\tau_\nu = \partial_\mu\tau_\nu - \Gamma^\lambda_{\mu\nu}\tau_\lambda, \tag{149}$$

and since $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$, the antisymmetrisation gives

$$F_{\mu\nu} = \partial_\mu\tau_\nu - \partial_\nu\tau_\mu - (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})\tau_\lambda = \partial_\mu\tau_\nu - \partial_\nu\tau_\mu. \tag{150}$$

Hence $F_{\mu\nu}$ carries no explicit dependence on the metric connection.

B.2 Variation of the kinetic term

Write

$$F_{\alpha\beta}F^{\alpha\beta} = F_{\alpha\beta}F_{\rho\sigma}g^{\alpha\rho}g^{\beta\sigma}. \tag{151}$$

Since $F_{\alpha\beta}$ is independent of $g^{\mu\nu}$,

$$\delta(F_{\alpha\beta}F^{\alpha\beta}) = 2F_\mu{}^\lambda F_{\nu\lambda}\,\delta g^{\mu\nu}. \tag{152}$$

Using $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}$, the kinetic variation becomes

$$\delta\Big(\sqrt{-g}\left[-\frac{1}{4}F^2\right]\Big) = \sqrt{-g}\left[-\frac{1}{2}F_\mu{}^\lambda F_{\nu\lambda} + \frac{1}{8}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right]\delta g^{\mu\nu}. \tag{153}$$

Thus,

$$T^{(F)}_{\mu\nu} = F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \tag{154}$$

B.3 Variation of the potential sector

Since $s = g^{\mu\nu}\tau_\mu\tau_\nu$,

$$\delta s = \tau_\mu\tau_\nu\,\delta g^{\mu\nu}. \tag{155}$$

Therefore,

$$\delta(\sqrt{-g}V) = \sqrt{-g}\left[V'(s)\tau_\mu\tau_\nu - \frac{1}{2}g_{\mu\nu}V(s)\right]\delta g^{\mu\nu}, \tag{156}$$

giving

$$T^{(V)}_{\mu\nu} = 2V'(s)\tau_\mu\tau_\nu - g_{\mu\nu}V(s). \tag{157}$$

B.4 Result

Combining both contributions,

$$T_{\mu\nu} = F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + 2V'(s)\tau_\mu\tau_\nu - g_{\mu\nu}V(s), \tag{158}$$

which reproduces Eq. (5) of the main text.

C Minkowski-space linear spectrum (ghost, speed, mass)

We derive the full linear mode spectrum of the torsion–phase field about a Minkowski background. The torsion-sector action is

$$S_\tau = \int d^4x\,\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(s)\right], \qquad F_{\mu\nu} \equiv \nabla_\mu\tau_\nu - \nabla_\nu\tau_\mu, \qquad s \equiv \tau_\mu\tau^\mu. \tag{159}$$

In Minkowski spacetime $g_{\mu\nu} = \eta_{\mu\nu}$ one has $F_{\mu\nu} = \partial_\mu\tau_\nu - \partial_\nu\tau_\mu$.

C.1 Background and quadratic expansion

We expand about a constant background $\bar{\tau}_\mu$,

$$\tau_\mu = \bar{\tau}_\mu + \delta\tau_\mu, \qquad \bar{\tau}_\mu = \text{const}, \qquad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1), \tag{160}$$

so that $\bar{F}_{\mu\nu} = 0$ and the background torsion magnitude is $\bar{s} = \bar{\tau}_\mu\bar{\tau}^\mu$.

Expanding the potential about $\bar{s}$,

$$V(s) = V(\bar{s}) + V'(\bar{s})\,\delta s + \frac{1}{2}V''(\bar{s})\,(\delta s)^2 + \mathcal{O}(\delta\tau^3), \qquad \delta s = 2\bar{\tau}^\mu\delta\tau_\mu + \mathcal{O}(\delta\tau^2), \tag{161}$$

the quadratic torsion Lagrangian becomes

$$\mathcal{L}_\tau^{(2)} = -\frac{1}{4}\,\delta F_{\mu\nu}\delta F^{\mu\nu} - \frac{1}{2}\,\delta\tau_\mu\,M^{2\,\mu\nu}\,\delta\tau_\nu, \tag{162}$$

with the effective mass matrix

$$M^2_{\mu\nu} = 2V'(\bar{s})\,\eta_{\mu\nu} + 4V''(\bar{s})\,\bar{\tau}_\mu\bar{\tau}_\nu. \tag{163}$$

The linearised field equation is therefore

$$\partial_\mu\delta F^{\mu\nu} - M^{2\,\nu}{}_{\mu}\,\delta\tau^\mu = 0. \tag{164}$$

C.2 Mode decomposition for a timelike background

For a timelike homogeneous background, choose a frame

$$\bar{\tau}_\mu = (\bar{\tau}_0, 0, 0, 0), \qquad \bar{s} = -\bar{\tau}_0^2. \tag{165}$$

Define the transverse and longitudinal mass scales

$$m_T^2 \equiv 2V'(\bar{s}), \qquad m_L^2 \equiv 2V'(\bar{s}) + 4\bar{s}\,V''(\bar{s}). \tag{166}$$

Decompose the spatial perturbations as

$$\delta\tau_i = \delta\tau_i^T + \partial_i\sigma, \qquad \partial^i\delta\tau_i^T = 0, \tag{167}$$

and work in Fourier space with $\partial_\mu \to (-i\omega, ik_i)$. The emergence of finite mass eigenvalues from the curvature of the torsion potential mirrors the geometric origin of the Yang–Mills mass gap obtained within the ECT framework [10].

C.3 Dispersion relations

After integrating out the non-dynamical constraint $\delta\tau_0$, the reduced equations give

Transverse sector:

$$\omega^2 = k^2 + m_T^2, \qquad \text{(two transverse modes)}. \tag{168}$$

Longitudinal sector:

$$\omega^2 = k^2 + m_L^2, \qquad \text{(one longitudinal mode)}. \tag{169}$$

Thus the torsion field propagates three physical degrees of freedom when $V'(\bar{s}) \neq 0$, all with luminal speed.

C.4 Ghost and tachyon conditions

Ghost-freedom requires positive kinetic energy, which in this Maxwell–Proca structure reduces to

$$V'(\bar{s}) > 0. \tag{170}$$

Tachyon-freedom requires

$$m_T^2 = 2V'(\bar{s}) > 0, \qquad m_L^2 = 2V'(\bar{s}) + 4\bar{s}\,V''(\bar{s}) > 0. \tag{171}$$

Equivalently,

$$V'(\bar{s}) + 2\bar{s}\,V''(\bar{s}) > 0. \tag{172}$$

Under (170) and (171), the Minkowski-space torsion perturbations are ghost-free, hyperbolic, and free of tachyonic growth.

C.5 Phase-locked extremum

If the background lies exactly at a phase-locked extremum $V'(\bar{s}) = 0$, the transverse modes become massless while the longitudinal sector depends on $V''(\bar{s})$ and may be absent or strongly constrained depending on the form of the potential.

D FRW linear perturbation spectrum (ghost, speed, mass)

We derive the linearised perturbation spectrum of the torsion–phase field on an FRW background and state the resulting ghost/gradient/mass stability conditions. The torsion sector is defined by

$$S_\tau = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(s) \right], \qquad F_{\mu\nu} \equiv \nabla_\mu \tau_\nu - \nabla_\nu \tau_\mu, \qquad s \equiv \tau_\mu \tau^\mu. \tag{173}$$

We consider a spatially flat FRW background

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \qquad \bar{\tau}_\mu = (\bar{\tau}_0(t), 0, 0, 0), \tag{174}$$

so that $\bar{F}_{\mu\nu} = 0$ and $\bar{s} = -\bar{\tau}_0^2$. The background field equation implies

$$V'(\bar{s}) = 0 \tag{175}$$

for a homogeneous phase-locked configuration.

D.1 SVT decomposition

We decompose metric perturbations in Newtonian gauge,

$$ds^2 = -(1 + 2\Phi)\, dt^2 + a(t)^2 (1 - 2\Psi)\, d\vec{x}^2 + 2a(t)\, B_i\, dt\, dx^i + a(t)^2 h_{ij}^{TT}\, dx^i dx^j, \tag{176}$$

with $\partial^i B_i = 0$, $\partial^i h_{ij}^{TT} = 0$, and $h^{TT\,i}{}_i = 0$. (Equivalent SVT choices are related by gauge transformations; the final reduced actions are gauge invariant.)

The torsion perturbations are decomposed as

$$\delta\tau_0 = \delta\tau_0, \qquad \delta\tau_i = \partial_i \delta\tau_L + \delta\tau_i^{(V)}, \qquad \partial^i \delta\tau_i^{(V)} = 0. \tag{177}$$

D.2 Quadratic action structure and constraints

Expanding the total action to second order yields a sum over tensor, vector, and scalar sectors,

$$S^{(2)} = S_T^{(2)} + S_V^{(2)} + S_S^{(2)}. \tag{178}$$

In all sectors, non-dynamical variables (those entering without time derivatives) act as constraints and may be integrated out. On the present FRW background, $\delta\tau_0$ is a constraint variable, and the metric potentials Φ and Ψ remain non-dynamical at leading order in the torsion sector (consistent with GR), though they mix with torsion scalars through the background $\bar{\tau}_0(t)$. Integrating out these constraints yields reduced actions for the propagating modes.

At the level of the quadratic action, the elimination of non–dynamical variables proceeds algebraically. In the vector sector, variation with respect to the metric vector B_i yields an elliptic constraint that fixes B_i in terms of the transverse torsion vector $\delta\tau_i^{(V)}$, which may then be substituted back into the action to obtain the reduced form (181). In the scalar sector, the lapse and shift perturbations (Φ, Ψ) and the temporal torsion component $\delta\tau_0$ enter without time derivatives. Varying the action with respect to these fields yields constraint equations that can be solved algebraically and substituted back, leaving a single gauge–invariant longitudinal torsion combination Ξ governing the reduced scalar dynamics (186).

D.3 Tensor sector

The tensor modes are carried by h_{ij}^{TT} and, on the phase-locked background (175) with $\bar{F}_{\mu\nu} = 0$, the torsion sector does not contribute a tensor kinetic term. The quadratic tensor action therefore retains the GR form,

$$S_T^{(2)} = \frac{M_{\rm Pl}^2}{8} \int dt\, d^3x\, a^3 \left[\dot{h}_{ij}^{TT}\dot{h}_{ij}^{TT} - \frac{1}{a^2}(\partial_k h_{ij}^{TT})(\partial_k h_{ij}^{TT})\right], \tag{179}$$

so gravitational waves propagate luminally with

$$c_T^2 = 1, \qquad m_T^2 = 0. \tag{180}$$

D.4 Vector sector

The vector sector contains the metric vector B_i and the transverse torsion vector $\delta\tau_i^{(V)}$. After integrating out the constraint B_i, the reduced quadratic action can be written as

$$S_V^{(2)} = \frac{1}{2} \int dt\, \frac{d^3k}{(2\pi)^3}\, a^3 \left[K_V\, |\dot{\delta\tau}^{(V)}|^2 - \left(\frac{c_V^2 k^2}{a^2} + m_V^2\right) |\delta\tau^{(V)}|^2\right], \tag{181}$$

where $\delta\tau^{(V)}$ denotes either of the two transverse polarisations. For the Maxwell–type kinetic term used here, one finds generically

$$K_V = 1, \qquad c_V^2 = 1, \tag{182}$$

while the effective vector mass is controlled by the curvature of the potential around the background,

$$m_V^2 = 2V'(\bar{s}). \tag{183}$$

On a phase-locked extremum with $V'(\bar{s}) = 0$, the transverse vector modes are massless:

$$m_V^2 = 0. \tag{184}$$

Vector-sector stability requires

$$K_V > 0, \qquad c_V^2 > 0, \qquad m_V^2 \geq 0. \tag{185}$$

D.5 Scalar sector

The scalar sector contains Φ, Ψ from the metric and $\delta\tau_0, \delta\tau_L$ from torsion. In Newtonian gauge, Φ and Ψ remain constraint variables, and $\delta\tau_0$ is also a constraint. After integrating out $(\Phi, \Psi, \delta\tau_0)$, the reduced scalar sector may be expressed in terms of a single gauge-invariant torsion-scalar combination (equivalently, the longitudinal mode) which we denote by Ξ (proportional to $\delta\tau_L$ up to background-dependent factors).

The reduced scalar action takes the standard form

$$S_S^{(2)} = \frac{1}{2} \int dt \, \frac{d^3k}{(2\pi)^3} \, a^3 \left[K_S \, |\dot{\Xi}|^2 - \left(\frac{c_S^2 k^2}{a^2} + m_S^2 \right) |\Xi|^2 \right]. \tag{186}$$

For the Maxwell–Proca structure, the kinetic and gradient coefficients are positive in the healthy branch, and one obtains luminal propagation,

$$K_S > 0, \qquad c_S^2 = 1. \tag{187}$$

The effective scalar mass is controlled by the second derivative of the potential around the background,

$$m_S^2 = 2V'(\bar{s}) + 4\bar{s} \, V''(\bar{s}). \tag{188}$$

In particular, for a phase-locked extremum with $V'(\bar{s}) = 0$,

$$m_S^2 = 4\bar{s} \, V''(\bar{s}) = -4\bar{\tau}_0^2 \, V''(\bar{s}). \tag{189}$$

Scalar-sector stability requires

$$K_S > 0, \qquad c_S^2 > 0, \qquad m_S^2 \geq 0, \tag{190}$$

i.e.

$$2V'(\bar{s}) > 0, \qquad 2V'(\bar{s}) + 4\bar{s} \, V''(\bar{s}) \geq 0. \tag{191}$$

On a strictly phase-locked background with $V'(\bar{s}) = 0$, the longitudinal stability condition reduces to

$$\bar{s} \, V''(\bar{s}) \geq 0 \qquad \Longleftrightarrow \qquad -\bar{\tau}_0^2 \, V''(\bar{s}) \geq 0. \tag{192}$$

If $V''(\bar{s}) = 0$ as well, the longitudinal sector becomes strongly constrained (degenerate) and the scalar mode may be absent.

D.6 Summary of FRW spectrum

On an FRW phase-locked background ($\bar{F}_{\mu\nu} = 0$ and $V'(\bar{s}) = 0$), the linear spectrum is:

- Tensor: two GR modes, $c_T^2 = 1$, $m_T^2 = 0$.
- Vector: two transverse torsion modes, $c_V^2 = 1$, $m_V^2 = 0$.
- Scalar: one longitudinal torsion mode with $c_S^2 = 1$ and $m_S^2 = 4\bar{s}\, V''(\bar{s})$, provided the scalar branch is non-degenerate.

Ghost/gradient/tachyon stability is ensured by (185) and (190).

References

[1] Hutchins, A.R., Hutchins, H.C. (2025). *A Geometrical Theory of Everything, Through Expansion, Contraction and Torsion (ECT) with Inverse-Square Laws Emerging from Geometric Principles'*. DOI: 10.5281/zenodo preprint.15408957. 3, 15

[2] Hutchins, A.R., Hutchins, H.C. (2025). *A Geometrical Theory of Gravity through Expansion, Compaction, and Torsion (ECT)*. Zenodo. DOI: 10.5281/zenodo preprint.16906263. 3, 4, 15, 16, 38

[3] Hutchins, A.R., Hutchins, H.C.,(2025) *ECT Algebra: A Minimal Three–Generator Lie Algebra and Its Representations*Zenodo2025, DOI: 10.5281/zenodo.17392122 4, 31

[4] Jacobson,T., and Mattingly, D., (2001) *Gravity with a dynamical preferred frame*, Phys. Rev. D **64**, 024028. DOI: 10.1103/PhysRevD.64.024028, arXiv: gr-qc/0007031. 4

[5] Heisenberg, L., (2014) *Generalization of the Proca Action*, J. Cosmol. Astropart. Phys. **05**, 015. DOI: 10.1088/1475-7516/2014/05/015, arXiv: 1402.7026. 4

[6] Armendáriz-Picón, C.,(2004) *Could dark energy be vector-like?*, J. Cosmol. Astropart. Phys. **07**, 007. DOI: 10.1088/1475-7516/2004/07/007, arXiv: astro-ph/0405267. 4

[7] Hehl, F.W., von der Heyde, P., Kerlick, G.D., and Nester, J.M.,(1976) *General Relativity with Spin and Torsion*, Rev. Mod. Phys. **48**, 393–416. DOI: 10.1103/RevModPhys.48.393. 4

[8] Hutchins, A.R., Hutchins, H.C. (2025). *Torsion Geometry in QED and QCD: A Unified ECT Framework*. Zenodo. DOI: 10.5281/zenodo preprint.16954624. 6, 31

[9] Hutchins, A.R., Hutchins, H.C.(2025) *The Wavefront of Light: A Geometric Field Encoding Framework for Matter Formation via ECT Shell Resonance*. Zenodo preprint, 2025 DOI: 10.5281/zenodo.15711603. 16

[10] Hutchins, A.R., Hutchins, H.C.,(2025) *Geometric Solution to the Yang–Mills Mass Gap Problem via ECT Framework,*Zenodo2025, DOI: 10.5281/zenodo.15427322 25, 46

[11] Hutchins, A.R., Hutchins, H.C. (2025). *Neutrino Masses and Oscillations as Torsion Shell Phase Drift in the Expansion–Compaction–Torsion (ECT) Framework'*. DOI: 10.5281/zenodo preprint.16929670. 30, 38

 Data Availability. No new data were generated for this study.

Author Contributions. Conceptual development, formal analysis, and manuscript preparation were carried out by the authors.

Funding. This research received no external funding.